

Pythagorean Linguistic Fuzzy Systems: Associative and Distributive Analysis of the \times_7 Operator toward Intelligent Scientific Applications

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Abstract: Fuzzy set theory provides a strong foundation for modelling vagueness and uncertainty in real-world problems. Intuitionistic fuzzy sets extended this idea by incorporating both membership and non-membership values. Later, Pythagorean fuzzy sets were introduced to capture more flexibility in decision-making scenarios. These developments led to the design of new aggregation operators, similarity measures, and distance functions that improved computational efficiency. Many studies have demonstrated the applications of these models in clustering, pattern recognition, service quality evaluation, and multi-criteria decision-making. Research on interval-valued and linguistic fuzzy approaches further enriched the decision-making process. In parallel, work on intuitionistic and complex intuitionistic fuzzy matrices emphasized normalization and Cartesian product operations. These contributions not only enhanced theoretical foundations but also widened the scope of practical applications. Building on these advancements, this study introduces a new innovation in the framework of linguistic sine trigonometric Pythagorean fuzzy numbers by extending Cartesian product operations \times_7 . Specifically, the following distributive and associative properties are established: $(\sin A \cup \sin B) \times_7 \sin C = (\sin A \times_7 \sin C) \cup (\sin B \times_7 \sin C)$, $(\sin A \cap \sin B) \times_7 \sin C = (\sin A \times_7 \sin C) \cap (\sin B \times_7 \sin C)$, $\sin C \times_7 (\sin A \cup \sin B) = (\sin C \times_7 \sin A) \cup (\sin C \times_7 \sin B)$, $\sin C \times_7 (\sin A \cap \sin B) = (\sin C \times_7 \sin A) \cap (\sin C \times_7 \sin B)$ and $(\sin A \times_7 \sin B) \times_7 \sin C = \sin A \times_7 (\sin B \times_7 \sin C)$. These properties demonstrate the structural soundness of the proposed operations and open new directions for applications in uncertainty-based modelling and decision-making.

Keywords: Set-theoretical distributive laws, Cartesian Product \times_7 , Sine–Trigonometric Pythagorean Fuzzy Linguistic Sets, Pythagorean fuzzy set.

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